THE DISCOVER INTERVIEW

SHING-TUNG YAU

The renowned mathematician explains how he charted the geometry of the universe and discovered the hidden dimensions of string theory.

BY **PAMELA WEINTRAUB** PHOTOGRAPHY BY **SHANNON TAGGART**

SHING-TUNG YAU is a force of nature. He is best known for conceiving the math behind string theory—which holds that, at the deepest level of reality, our universe is built out of 10-dimensional, subatomic vibrating strings. But Yau's genius runs much deeper and wider: He has also spawned the modern synergy between geometry and physics, championed unprecedented teamwork in mathematics, and helped foster an intellectual rebirth in China.

Despite growing up in grinding poverty on a Hong Kong farm, Yau made his way to the University of California at Berkeley, where he studied with Chinese geometer Shiing-Shen Chern and the master of nonlinear equations, Charles Morrey. Then at age 29 Yau proved the Calabi conjecture, which posits that six-dimensional spaces lie hidden beneath the reality we perceive. These unseen dimensions lend rigor to string theory by supplementing the four dimensions—three of space and one of time—described in Einstein's general relativity.

Since then Yau has held positions at the Institute for Advanced Study, Stanford University, and Harvard (where he currently chairs the math department), training two generations of grad students and embarking on far-flung collaborations that address topics ranging from the nature of dark matter to the formation of black holes. He has won the Fields Medal, a MacArthur Fellowship, and the Wolf Prize.

Through it all, Yau has remained bluntly outspoken. In China he has called for the resignation of academia's old guard so new talent can rise. In the United States he has critiqued what he sees as rampant errors in mathematical proofs by young academics. Yau has also strived to speak directly to the public; his book *The Shape of Inner Space,* coauthored with Steve Nadis, is scheduled for publication this fall. He reflected on his life and work with DISCOVER senior editor Pamela Weintraub at his Harvard office over four days in February.

You've described your father as an enormous intellectual influence on you. Can you tell me about him?

He went to Japan to study economics, but he came back to help the Chinese defend themselves before the Japanese invaded in 1937. By the end of the war he was distributing food and clothes to the poor for the U.N. After the revolution in 1949, he worried about getting in trouble with the Communists, so he brought the whole family to Hong Kong. We were very poor—at first we were almost starving—but my father had a large group of students constantly at home to talk about philosophy and literature. I was 10, 11, 12 years old, and I grew accustomed to abstract reasoning. My father made us memorize long essays and poems. At the time I didn't understand what they meant, but I remembered them and later made use of it.

Did part of you ever rebel?

I read most of the *Kung Fu* novels in secret. I quit school for more than half a year. I'd wake up and say I was going, but I'd spend the whole day exploring the mountains and then come back—but I did the homework that my father assigned to me at home.

I heard you led a gang at one point.

I had a group of friends under me. I'd go around, and sometimes we ended up in fistfights with some other groups. So?

How did you go from that rough-and-tumble young man to the focused person you are now?

In the early 1960s my father was chairman of the department of literature and philosophy at Hong Kong College. The college president wanted to make a deal with the Taiwanese government to

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send in spies. My father refused to go along and resigned. That created a big money problem because he had eight children by then. My father had to run around among different, distant colleges to support the family. Back in China he'd lent a friend some money, and after the Communists took over, the friend moved to Macau, a city near Hong Kong, and ran his own schools. So he told my father, "I cannot return your money, but your daughter can come to my school, and I'll give her free room and board and free tuition." So my older sister went to Macau to study and got some flu, some funny disease, we never knew exactly what. She came back and she was treated, but she died in 1962. Then my elder brother got a brain disease; at the time we didn't know what it was. My father had all kinds of burdens on his shoulders and then *he* got a disease, which I believe was cancer, but we didn't know much in those days. My mother was running around trying to get funding to help my father. Finally we raised some money, but it was too late. He died after two months in the hospital in 1963, in the middle of my studies in the ninth grade. We could no longer afford our apartment, so we were kicked out. That's when I realized I would have to make decisions for myself.

What did you do then?
After a while the government leased us some land, and we built a small house thanks to money from friends, but it was in a village far from school. The other kids looked down on us for being poor, and I had to ask the school president to allow me to pay tuition at the end of the year, when my government fellowship came through. It was humiliating. But I studied hard and did very well, especially in math. Then a former student of my father started a primary school in a town closer to school. He said I could help teach math and stay there at night. I had to take care of myself, I had to wash things and all of that, but I learned how to survive.

What happened once you made your way to college?
I had fallen in love with math early on, but at the Chinese University of Hong Kong I realized that mathematics was built on standard actions and logic. Soon I had arranged to take tests for the required math courses without actually attending while sitting in on more advanced classes, and no one seemed to mind. In my second year, Stephen Salaff, a young mathematician from U.C. Berkeley, came to teach in Hong Kong. He liked to talk to the students in the American way: He gave lectures and then he asked students questions. In many cases it turned out I could help him more than he helped me, because there were problems he couldn't solve during class. Salaff suggested I apply to graduate school early. I was admitted to Berkeley and even got a fellowship. I borrowed some money from friends and flew to San Francisco in September 1969.

What did you think of California when you arrived?
The first thing that impressed me was the air. In Hong Kong it's humid, hot, but in California it was cool and clear. I thought it was like heaven. A friend of Salaff's came to the airport to pick me up and took me to the YMCA, where I shared a big room with four or five people. I noticed that everybody was watching baseball on TV. We didn't have a TV at home. My neighbor who was sleeping there was a huge black man. He was talking in a language I had never heard before. He said, "Man, where the hell you come from?" It was fun, but I had to look for an apartment. I was walking around the street when I met another Chinese student from Hong Kong and we decided to share, but we couldn't afford a place. We looked around and found another Chinese student, from Taiwan, so there's three of us and it's still not enough. Then we found an Alaskan also studying math, also on the street. So four of us went in together and the rent for each was $60 a month. My fellowship gave me $300 a month, and I sent half of it home.

What about your math studies?
There were many holes in my knowledge so I'd wake up early and start class at 8 a.m. I took three classes for credit, and the rest I audited. I brought my own lunch so even at lunchtime I was in class. I was especially excited about topology because I thought it could help reveal the structure of space. Einstein used geometry in his equation to give us the local picture: how space curved around our solar system or a galaxy. But the Einstein equation didn't give the overall picture, the global structure of the whole universe. That's where topology came in.

What is topology? Is it like geometry?
Geometry is specific and topology is general. Topologists study larger patterns and categories of shapes. For example, in geometry, a cube and a sphere are distinct. But in topology they are the same because you can deform one into the other without cutting through the surface. The torus, a sphere with a hole in the middle, is a different form. It is clearly distinct from the sphere because you cannot deform a torus into a sphere no matter how you twist it.

Does that mean geometry and topology are really two perspectives on the same thing?
Yes. It is like Chinese literature. A poem might describe a farewell between lovers. But in the language of the poem, instead of a man and woman, there is a willow tree, where the leaves are soft and hanging down. The way the branch is hanging down is like the feeling of the man and the woman wanting to be together. Geometry gives us a structure of that willow tree that is solid and extensive. Topology describes the overall shape of the tree without the details—but without the tree to start with, we would have nothing.

It has always amazed me to observe how different groups of people look at the same subject. My friends in physics look at space-time purely from the perspective of real physics, yet the general theory of relativity describes space-time in terms of geometry, because that's how Einstein looked at the problem.

When you looked at the world through the lens of geometry and topology, what did you learn?
That nonlinear equations were fundamental because in nature, curves abound. Climate isn't linear. If the wind blows stronger that way, it may cause more trouble over there; it may even depend on the geometry of the earth. Usually you see the stock market described by linear equations and straight lines, but that is not really correct. The stock market fluctuates up and down in a nonlinear way. The Einstein equation described the curvature of the universe, and it was nonlinear. I ended up learning nonlinear equations from a master, although I didn't know he was a master at the time. His name was Charles Morrey, and he was a classical gentleman. He always dressed in suits in class. He was a very nice

man. Even if I was the only one there, he would lecture to me, just as if he were lecturing to the whole class.

Wait—you were sometimes the only one in his class?
Why should people care about ancient days? Morrey didn't use modern notations. His book was hard to read. Kent State had just happened. The students and the faculty were all on strike, but Morrey still gave lectures. Soon everyone had dropped the class but me.

What happened next in your mathematical explorations?
It was Christmas break and I couldn't go home, so I spent my time at the library reading all the journals and looking at rare books. That was the first time I met my wife, although only much later were we formally introduced. Through all this reading about topology I came across a theorem that talked about loops where curvature is everywhere negative—where the curve goes in like a saddle. The theorem says that when we have two such loops with vertices at the same point, they can't deform into each other by bending or twisting unless they are equal to or multiples of each other. I came up with a related theorem: If the curvature is either negative or zero and if the loops conform, then there must be a lower-dimensional surface—specifically a torus—sitting somewhere inside.

How can a lower dimension sit inside a higher one?
Imagine attaching a rubber band to the handle of a coffee cup. The cup has three dimensions, but the rubber band, which is just a curved line, effectively has only one.

Why should anyone other than a mathematician care about a torus or a string hidden within higher dimensions?
Because topology can affect and constrain geometry in the physical world. If water flows around a sphere, for example, there must be two points where the water is totally still. On a planet covered with an ocean, the water can't all flow in the same direction, say east to west, everywhere, without hitting a snag. In the case of another topology, the torus, water can flow around and around and there's no point at which the flow stops because the hole eliminates the impasse. For each fixed topology, the geometry follows different laws.

In other words, you realized that topology sets the basic rules for geometry, which in turn affects the world around us. But then you went further, asking whether the underlying structure of space might explain the laws of physics. How so?
I started to look into complex manifolds. A manifold is just a space, with each point immediately around you looking like Euclidean space—the familiar kind of space that we see around us. Imagine the earth is covered with a checkerboard or a grid, like latitude and longitude. This is the kind of coordinate system that Descartes introduced to geometry in the 17th century. At each point on the grid the space appears flat and finite, but it's actually curved, a sphere. Instead of being measured with real numbers, though, we measure complex manifolds with complex numbers, in which one of the coordinates includes a real number multiplied by the square root of negative 1—an imaginary number that we call *i.* [Since the product of two negative numbers is positive, ordinary math suggests the square root of negative 1 cannot exist—hence the moniker "imaginary."]

How can complex manifolds and complex numbers help us understand the structure of space?

Space is not necessarily something you see in day-to-day life. You can define geometry locally, but globally you cannot visualize the big picture, you can only imagine it and represent it through coordinates. We draw lines of latitude and longitude in a coordinate system for the continents. But that system doesn't work well at the north or south poles, where all the lines converge. In order to get a more complete picture in those regions, we need another, more localized coordinate system for more detail. In the end, we need several such coordinate systems patched together to get a detailed picture of the entire globe.

More generally, in describing any space, we are not restricted to the three dimensions we experience in our lives. Mathematically, we can suggest any number of dimensions: two, three, four, five, ten, just by drawing additional coordinates on a grid. In complex space, every number in a coordinate system describes not just one dimension but two. Most important, complex numbers make it simpler to move from one coordinate system to another, a necessary step when working in the higher dimensions necessary for string theory.

You are best known for your work on the Calabi conjecture, which at the time was a major unsolved problem in higher-dimensional mathematics. What attracted you to it?
I was drawn to important problems that gave insight into geometry and space-time. Sometimes solving a problem creates a new kind of thinking, sometimes the math itself is beautiful. The problem I went with, the Calabi conjecture, is a very elegant statement about the curvature of complex manifolds.

What does "curvature" mean in this context, since you aren't talking about the kind of curves we normally experience?
Curvature is second-order information—for instance, suppose I am driving a car around a curved freeway. The car's velocity will change as

If these spaces modeled the six-dimensional space called for in string theory, they would help us deduce the geometry and the physical laws of the universe.

you go, so you can measure the curve in terms of changes in velocity along that one-dimensional line. Then there is Gauss curvature, which gives you the curvature of a two-dimensional surface by multiplying the largest and smallest curvature of the family of all curves tangential to the surface at a given point. For higher-dimensional space, such as the three-dimensional space around us, we calculate the curvature of all the two-dimensional surfaces passing through the point where curvature is in question. Finally there is Ricci curvature, which we measure by averaging the curvature of all two-dimensional surfaces tangential to each other along a common direction. In essence, Ricci curvature is an average of part of the total curvature of a space. It is an abstract geometric concept, but it is fundamental.

Why is Ricci curvature fundamental?
In physics, Ricci curvature is analogous to matter. Space with zero Ricci curvature is space without matter—a vacuum.

And how does all this relate to the Calabi conjecture?
Calabi said that certain topological conditions call for the existence of nonflat, closed, complex spaces without Ricci curvature anywhere. Such spaces would enjoy many beautiful properties. You might find the sub-dimensional loops or the torus I described in that very first paper I wrote—or you might find intersecting branes [short for "membrane," another topological shape]. I was 100 percent sure that the spaces Calabi called for could not exist. No mathematician or physicist had ever found an example of one, and most geometers considered them too good to be true.

So what did you do next?
I spent a lot of time thinking about how to disprove Calabi. By 1973 I was teaching at SUNY–Stony Brook and planning to move to Stanford. That May I put my belongings into this little Volkswagen and drove across the country on Highway 80. I thought America was a country where everyone traveled around, but to my amazement, a lot of the people I met told me they had never driven more than 10 miles from their town. I crossed the Rocky Mountains. The car broke down at one point. By the time I was at Stanford for a few months, I thought I had finally proven Calabi wrong.

Disproving the Calabi conjecture would have been a major achievement; how did you announce it?
In August there was a big conference at Stanford with the top geometers in the world, including Calabi. I talked to Calabi and told him my idea. He said, "That sounds great. Why don't you give a discussion about it to me?" It was scheduled for 7 p.m. Calabi brought a few colleagues from the University of Pennsylvania, and then a few others heard about it, and a few others. There was a little crowd. I talked for about an hour, and Calabi was excited. "I've been waiting for this for a long time, and I hope it's right," he said. All the other people said, "Great, finally we can stop the wishful thinking that Calabi is true." Then Calabi wrote to me in October. He said, "I'm trying to reconstruct your argument, and I'm having some difficulty. Could you explain the detail?" I started to reconstruct it and I found a problem too. I got totally embarrassed. I did not respond to Calabi at that moment and instead tried extremely hard to patch up the proof. I couldn't, so I looked around to find other examples where Calabi was wrong. I didn't sleep for two weeks. But every time I found an example that was close, the proof fell apart at the last minute. Finally I said, gee, this cannot be such a delicate matter. Now I had much deeper insight into the issue and felt there must be some truth to the whole thing. I determined that it had to be right.

So after all that work trying to prove that Calabi's conjecture was wrong, you decided it was correct after all?
I began developing the tools to understand it, and by 1975, only one part of the proof was left. That year my wife got a job in Los Angeles. I moved to UCLA. All in a short time, we got married, bought a car, bought a house in the Valley, and had to look for furniture. My mother came from Hong Kong for the wedding, and then her parents came—they all stayed under one roof and got into fights; it was complicated and crazy. I was fed up, so I locked myself in the study and thought about Calabi instead of the family problems, and I solved the whole thing. I went over the proof three times in detail, and I went to see Calabi in Pennsylvania. On a snowy Christmas Day, he came with me to visit mathematician Louis Nirenberg at New York University. We spent all day Christmas going over it, and I spent the next month writing up the proof for publication.

The implications were enormous. You were instantly famous.
It solved some major problems in algebraic geometry—about a dozen of them. A lot of people offered me jobs.

Some of the higher-dimensional spaces now called Calabi-Yau spaces proved fundamental to string theory. What is the connection?
When Einstein published his general theory of relativity in 1915, there was an immediate urge to unify the force of gravity with the other forces known at the time, with electricity and magnetism. Mathematicians thought they could do this with five dimensions, four of space and one of time. But then physicists found new particles and needed extra dimensions for the strong force and the weak force. When they worked it all out, they determined they could explain the universe with something they called string theory, which replaces the pointlike particles in particle physics with tiny, elongated vibrating strings. To be consistent with quantum theory, the strings needed 10 dimensions in which to vibrate: three of space, one of time, and six dimensions of "compact space." Dimensions in compact space are so small you can't detect them through any conceivable experiment. They amount to pure structure. It so happens that Calabi-Yau spaces with six dimensions also have specific topological traits corresponding to the requirements of string theory. If these spaces truly modeled the six-dimensional space called for in string theory, they would help us deduce the geometry and, by extension, the physical laws of the universe.

Some cosmology theories imply the existence of other universes. Could each Calabi-Yau space describe a different set of laws in each of those universes?
Yes, each isolated universe can be modeled by a different Calabi-Yau space. But some of my colleagues have also studied a beautiful concept called mirror symmetry, in which each space has a mirror image with the same quantum field theory and the same physics.

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How many Calabi-Yau spaces are there?
Using a computer program, Philip Candelas at the University of Texas at Austin found up to 10,000 Calabi-Yau spaces, with almost half of them mirror partners of each other. Each member of a pair is topologically distinct but still conforms to the other algebraically and gives rise to the same forces, the same particles, the same rules. The resulting geometric structure can be used to determine physical quantities associated with each space, like particle mass.

String theory is often described as a mathematically elegant way to explain all of physics. But how can we know that it describes the real universe?
We cannot know for sure, but the mathematics inspired by string theory solves some old, longtime questions. That part is rigorous and its truth cannot be challenged. If the structure of the math is deep, it will solve something in nature one way or another; it is difficult to imagine that such deep structure corresponds to nothing. Everything fundamental in math has ultimately had a meaning in the physical world.

You've long promoted mathematics in China. How have academic conditions changed there over the years?
I first went back in 1979, right after China opened up to outsiders. People were poor. Times were difficult. It was bedlam. I saw lots of people who were uneducated, and I felt I needed to help. By 1985 I'd taught about 15 Chinese grad students accepted to programs in the States. At first it was my adviser and mentor, Shiing-Shen Chern, who went to China and founded a mathematical institute there. I didn't want to interfere with his work, but he was getting old and I started to go visit more often. In 1994 I was asked to give a speech. I said it's great that China has an open policy; now we must start moving forward step by step, training young people to establish an intellectual base.

Eventually you founded four math institutes in China. How did that happen?
I met with Jiang Zemin, the future president, who wanted me to help build up math in China. After that, with the help of a donor, I built the Institute of Mathematical Sciences at the Chinese University of Hong Kong followed by three more institutes on the mainland—but China has always been run in a collaborative way, and other universities began demanding part of the funds. Still, the institutes have been able to carve out some independence, and today I go to China five or six times a year.

In the past decade, you've been critical of science and math in China. Why?
The university system is beset by academic politics, and it's difficult for young people to move ahead. When China opened up, the people running things were in their fifties and sixties. The same people are still running things. Most do not follow modern developments because of their age. There are some brilliant young people, but it is a struggle for them to be recognized. Often that happens in China only after they are recognized by the outside world. I said, "Give some freedom to the young guys," and people got upset.

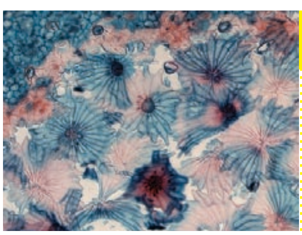
You've commented that at the highest levels of accomplishment, Chinese mathematicians have far to go, and that the best of them have left the country. What are the prospects for math and science in China today?
The economy has been getting better and the government wants to invest more in science, so in the long run, I think the future is bright. Many more Chinese graduate students who come to study in the United States will be willing to return to China.

How does China's relationship with the United States come into play?
I see a constructive relationship for academia. The U.S. gets human resources in the form of bright, young Chinese kids. The students learn well here, because the U.S. provides them with the freedom to research in their own way, and some of them will bring their knowledge back to China. But my goal is to train many more young mathematicians within China by providing an environment that allows them to focus on research and be recognized for their work.

You have criticized the academic system in the United States as well.
Young people are under too much pressure here. As a result, some of the proofs they publish are factually wrong. Before I published my proof of the Calabi conjecture, I checked it three times. Many young mathematicians don't do that.

Most people don't realize how political math can be: In 2006 *The New Yorker* accused you of taking credit from the Russian mathematician Grigory Perelman after he proved the famed Poincaré conjecture. What happened there?
In a process as intricate and daunting as proving the Poincaré conjecture, it is understandable that Perelman released his manuscript with several key steps merely sketched or outlined. One of my students tried to fill in some of the details, and I supported that. I also said that my friend Richard Hamilton, a geometer at Columbia University, laid much of the groundwork that Perelman ultimately relied on to construct his proof. For these things *The New Yorker* tried to accuse me of stealing credit, but that is ridiculous. What I think of as the Hamilton-Perelman proof of the Poincaré conjecture is a great triumph for mathematics, and I fully support the award of the Fields Medal to Perelman. Hamilton deserved the Fields Medal too, but he was ineligible because of the age restriction [you must be under 40]. To suggest that my position has ever been any different is completely untrue.

Physicists often talk about the beauty of math. What does that mean to you?
The first time I saw my wife, I thought she was charming—more than charming, shocking to me. I had great motivation to know her more. When I look at the Calabi conjecture, it shocks me too. It's an elegant, simple construct and explains a great deal. It's exciting when you go deeper and deeper into a complicated structure that you can spend most of a lifetime working on. It was shocking when it showed up in physics, and it's beautiful whether it's true or not.



MAY'S **WHAT IS THIS? OAK LEAF**

The Northern Hemisphere is home to about 450 species of oak. This micrograph image of an oak leaf's surface reveals bunches of tiny hairs called trichomes, which concentrate light for photosynthesis and reduce water loss. Oak leaves also contain tannins, which can poison horses that have a taste for foliage.